Stochastic resonance in a system of ferromagnetic particles

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We show that a dispersion of monodomain ferromagnetic particles in a solid phase exhibits stochastic resonance when a driven linearly polarized magnetic field is applied. By using an adiabatic approach, we calculate the power spectrum, the distribution of residence times, and the mean first passage time. The behavior of these quantities is similar to the behavior of corresponding quantities in other systems where stochastic resonance has also been observed.

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I. INTRODUCTION

The phenomenon known as stochastic resonance (SR) was first predicted by Benzi *et al.* [1] and consists of the coherent response of a multistable stochastic system to a driven periodic signal. Up to now, SR has been observed in diverse physical situations as in lasers, in electron paramagnetic resonance, or in free standing magnetoelastic beams. The description of the phenomenon as well as its fundamentals and applications are included in the recent reviews by Moss [2] and Wiesenfeld and Moss [3] (see also Refs. [4] and [5]).

Our purpose in this paper is to show theoretical predictions about the occurrence of the phenomenon in a system of ferromagnetic monodomain particles dispersed in a solid phase (a crystalline polymer, for example) when an alternating magnetic field is imposed. In a ferromagnet the interdomain walls are of the order 10^{-6} cm, so particles whose size is of this order of magnitude or less may be considered monodomains [6]. Such particles are always magnetized to the spontaneous magnetization M_s. For these fine ferromagnetic particles the energy consists of contributions coming from two competing mechanisms that tend to orient the magnetic moment: the potential energy of the field and the energy of anisotropy. The energy is therefore a nonlinear function of the orientation angle, which is precisely the stochastic variable. This fact was already taken into account by Néel [7] to estimate the relaxation time for the magnetic moment in magnetic powders. As we will show, under certain conditions the ferromagnetic particle constitutes a bistable stochastic system, with the external field providing a periodic contribution.

The stochastic behavior of systems of ferromagnetic particles was discussed in Ref. [8], where a Fokker-Planck equation for the probability density of the orientations of the particles was derived. This equation is related to the Landau-Gilbert equation [9,10] in which a stochastic source accounting for Brownian motion of the magnetic moment is added. The procedure is restricted, however, to the case in which the external magnetic field is constant in time. This theory provides the framework for our subsequent analysis and consequently must be extended to the case of a time-dependent magnetic field.

We have organized the paper in the following way. In Sec. II we introduce our model and from the Gilbert-Landau equation we establish the kinetic equation for the probability distribution of the magnetic moment. In Sec. III, by using an adiabatic approximation, we compute the power spectrum of the fluctuations of the magnetic moment, while, in Sec. IV, and within the framework of the same approach, we calculate the probability distribution of residence times and the mean first passage time. Finally, in Sec. V we give numerical values of the characteristic parameters of our system and discuss our main results. Additionally, we show that because of the very short time scale that rules the relaxation of the system, the adiabatic approach is justified.

II. DISPERSION OF FERROMAGNETIC PARTICLES

We consider an assembly of single-domain ferromagnetic particles dispersed in a solid phase at a concentration that is assumed to be low enough to avoid magnetic interactions among them. When we apply an external, uniform, ac magnetic field $\vec{H}(t) = \vec{H}_0 \sin\omega_0 t$, $\vec{H}_0$ being the magnetic field strength and ω_0 its angular frequency, the energy of each particle splits into contributions coming from the external field and the crystalline anisotropy [6] and is given by

$$U(t) = -\vec{m}\cdot\vec{H}(t) + K_a V_p (\hat{m}\cdot\hat{s})^2 \ . \tag{1}$$

Here $\vec{m} = m_s\hat{m}$ is the magnetic dipole moment; $m_s = M_s V_p$ is the magnetic moment strength, with M_s the saturation magnetization and V_p the volume of the particle; $K_a > 0$ is the anisotropy constant; and $\hat{s}$ is a unit vector perpendicular to the symmetry axis of the particle.

The dynamics of the magnetic moment $\vec{m}$ is governed by the Gilbert equation [10,11]

$$\frac{1}{\gamma}\frac{d\vec{m}}{dt} = \vec{m}\times(\vec{H}_{\rm eff} + \vec{H}_d) \ , \tag{2}$$

where γ $(= -e/mc)$ is the gyromagnetic ratio. From (2) one may identify the two mechanisms responsible for the variation of $\vec{m}$. The effective field

$$\vec{H}_{\text{eff}} \equiv -\frac{\partial U}{\partial \vec{m}} = \vec{H}(t) - 2\frac{K_a V_p}{m_s}(\hat{m}\cdot\hat{s})\hat{s}, \tag{3}$$

which implies a Larmor precessional motion of $\vec{m}$ and the mean field

$$\vec{H}_d \equiv -\eta\frac{d\vec{m}}{dt}, \tag{4}$$

which introduces a damping whose microscopic origin lies in the collisions among the electrons participating in the formation of the magnetic moment of the domain. In Eq. (4) η is a damping coefficient.

Equation (2) can be solved self-consistently giving

$$\frac{d\vec{m}}{dt} = \vec{\omega}_L \times \vec{m} + h\vec{m}\times\vec{H}_{\text{eff}}\times\vec{m} \ , \tag{5}$$

where $\vec{\omega}_L = -m_s g\vec{H}_{\text{eff}}$ is the Larmor angular frequency of the precessional motion executed by the magnetic moment of a dipole. Moreover, we have introduced the quantities

$$g = \frac{\gamma}{m_s(1+\eta^2 m_s^2\gamma^2)} \tag{6}$$

and

$$h = -\frac{\eta\gamma^2}{(1+\eta^2 m_s^2\gamma^2)} \ . \tag{7}$$

When the external field is constant, the precessional motion is extinguished in a time scale $\tau_0 = (m_s h H_{\text{eff}})^{-1}$, obtained by a comparison of the left-hand term and the second right-hand term in (5). Thus when Brownian motion is absent, the $\vec{m}$ become parallel to $\vec{H}_{\text{eff}}$ for times larger than τ_0. Keeping only first-order terms in the damping coefficient, one obtains the Landau equation [9]

$$\frac{d\vec{m}}{dt} = -\gamma\vec{H}_{\text{eff}}\times\vec{m} + \lambda\vec{m}\times\vec{H}_{\text{eff}}\times\vec{m} \ , \tag{8}$$

where λ may be identified as $\eta\gamma^2$.

The presence of thermal noise was considered by Brown [8] by simply adding the random field $\vec{H}_r$ to the Gilbert equation (2). This equation thus becomes a nonlinear Langevin equation with multiplicative noise

$$\frac{1}{\gamma}\frac{d\vec{m}}{dt} = \vec{m}\times(\vec{H}_{\text{eff}} + \vec{H}_d + \vec{H}_r) \ . \tag{9}$$

The random term constitutes a Gaussian stochastic process with zero mean and a fluctuation-dissipation theorem given by

$$\langle \vec{H}_r(t')\vec{H}_r(t'+t)\rangle = 2k_B T\eta\vec{\vec{1}}\delta(t) \ , \tag{10}$$

where $\vec{\vec{1}}$ is the unit tensor.

Following the standard procedure, it is possible to derive the Fokker-Planck equation related to (9). One then obtains [8,12]

$$\frac{\partial\psi}{\partial t} - \vec{S}\cdot\vec{\omega}_L\psi = -\frac{1}{2\tau}\vec{S}\cdot\psi\vec{S}\left(\frac{U(t)}{k_B T} + \ln\psi\right) \ , \tag{11}$$

where $\psi(\hat{m},t)$ is the distribution function for the orientations of the vector $\hat{m}$ and $\vec{S} = \hat{m}\times\frac{\partial}{\partial\hat{m}}$ is the rotational operator. From this equation we infer the appearance of the relaxation time $\tau = (-2k_B T h)^{-1}$, corresponding to the time scale in which one achieves the stationary state where the probability flux is constant.

If the external magnetic field is applied along the direction of the easy axis of magnetization, the problem posed by Eq. (11) has an axial symmetry. In this case, the energy of the system can be written as

$$U = -m_s H_0 \sin\omega_0 t\cos\theta + K_a V_p \sin^2\theta \ . \tag{12}$$

Therefore, it turns out that for $H_0 < H_c$, with H_c $(= 2K_aV_p/m_s)$ a critical field, the system is bistable. This critical field coincides with the coercive field of ferromagnets. In view of (12) the potential energy has two minima for the values of the angle 0 and π denoted by θ_+ and θ_-, respectively, and one maximum at the angle θ_m, which is determined from the condition

$$\cos\theta_m = -\frac{H_0}{H_c}\sin\omega_0 t \equiv -\varepsilon(t) \ . \tag{13}$$

Given that $H_0/H_c < 1$, one has the inequality $0 < \theta_m < \pi$. The energy at the three singular points is then expressed as

$$U_\pm \equiv U(\theta_\pm) = \mp m_s H_c \varepsilon \tag{14}$$

and

$$U_m \equiv U(\theta_m) = 1/2 m_s H_c(1+\varepsilon^2) \ . \tag{15}$$

Note that the phase difference between U_+ and U_- is equal to π and that the position of the maximum θ_m oscillates in phase with U_+. In Fig. 1 we have represented

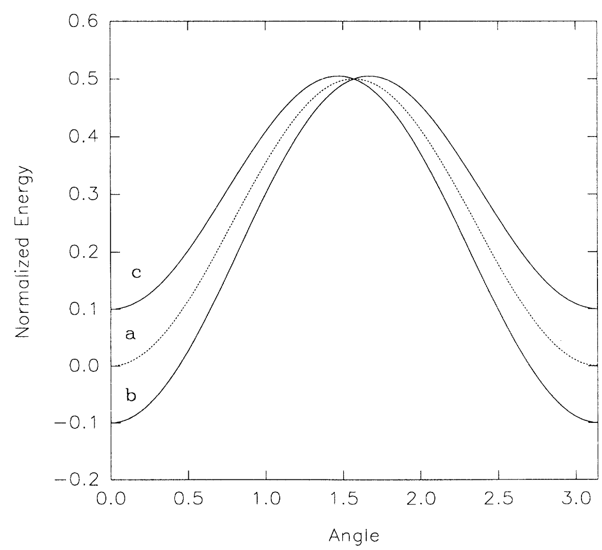

FIG. 1. Normalized energy U/m_sH_0 for (a) $\omega_0 t = 0$, (b) $\omega_0 t = \pi/2$, and (c) $\omega_0 t = 3\pi/2$.

the potential given through Eq. (12) in a cycle of the external magnetic field. As follows from the figure, the effects of the modulation are to vary the relative depth of the two wells of the potential and also to change the barrier height, which corresponds to additive and parametric modulation, respectively [2]. For the particular case we have considered, Eq. (11) becomes

$$2\tau\frac{\partial\psi}{\partial t}=\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left\{\sin\theta\left(\frac{1}{k_BT}\psi\frac{\partial U}{\partial\theta}+\frac{\partial}{\partial\theta}\psi\right)\right\} \quad . \tag{16}$$

This equation will constitute our starting point in the analysis of the stochastic resonance in the following sections.

III. POWER SPECTRUM

Our purpose in this section is to compute the power spectrum of the fluctuations of the angle θ, which, according to the Wiener-Khinchin theorem, is related to the intensity of the fluctuations. To this end we will first proceed to introduce some assumptions that imply a reinterpretation of the stochastic process $\theta(t)$. When $t \gg \tau$ (a situation in which the system has reached its stationary state) and for $(U_m - U_\pm)/k_BT$ high enough, we can reformulate the problem in terms of the variable $x \equiv \cos\theta$, assumed to be discrete, taken on the values $x_\pm = \pm 1$. The corresponding probability distribution is then given by the function

$$p(x,t)=n_+(t)\delta_{x,x_+}+n_-(t)\delta_{x,x_-} \quad , \tag{17}$$

where

$$n_+(t)=1-n_-(t)=\int_0^{\theta_m}\psi(\theta,t)\sin(\theta)d\theta \quad . \tag{18}$$

In these equations $n_\pm$ are the probabilities to find the magnetic moment around the minima $\pm$ at time t. Therefore, $n_\pm$ can be interpreted as the fractional population at the minima $\pm$. From the Fokker-Planck equation it is then possible to derive the rate equations [13,14] for these variables. They read

$$\begin{aligned}\frac{d}{dt}n_+ &= -\frac{d}{dt}n_- = W_-n_- - W_+n_+ \\ &= -(W_-+W_+)n_+ + W_- \ ,\end{aligned} \tag{19}$$

where W_+ and W_- are the transition rates corresponding to the jumps $(+) \longrightarrow (-)$ and $(+) \longleftarrow (-)$, respectively. These transition rates were computed by Brown in the case of a constant external field [8]. He obtained

$$W_\pm = c_\pm\exp\left[-(U_m-U_\pm)/k_BT\right] \quad , \tag{20}$$

with

$$c_\pm = hk_\pm(k_m/2\pi k_BT)^{1/2}\sin\theta_m \quad , \tag{21}$$

where

$$k_+ \equiv \frac{d^2U}{d\theta^2}(\theta=0)=m_sH_c(1+\varepsilon) \quad , \tag{22}$$

$$k_- \equiv \frac{d^2U}{d\theta^2}(\theta=\pi)=m_sH_c(1-\varepsilon) \quad , \tag{23}$$

and

$$k_m \equiv m_sH_c(1-\varepsilon^2) \quad . \tag{24}$$

In this case, one may identify from Eq. (19) the relaxation time to the equilibrium state between the wells, i.e., when the probability current is zero. This time scale, referred to as the Néel relaxation time [7,8], is defined as $\tau_N = (W_+ + W_-)^{-1}$.

The extrapolation of Eqs. (20)–(24) to the case concerning us is accomplished by assuming $\varepsilon(t)$ as given by Eq. (13). As obtained by McNamara and Wiesenfeld [15], the solution to the rate equations is found to be

$$n_\pm(t)=g^{-1}(t)\left[n_\pm g(t_0)+\int_{t_0}^{t}W_\mp(t')g(t')dt'\right] \quad , \tag{25}$$

with

$$g(t)=\exp\left[\int_{-\infty}^{t}\left[W_+(t')+W_-(t')\right]dt'\right] \quad . \tag{26}$$

In order to compute $n_\pm$ we perform a Taylor expansion of the transition rates with respect to the parameter ε. Up to the second order in this quantity we obtain

$$W_\pm = \frac{1}{2}(\alpha_0+\alpha_1p_\pm\varepsilon+\alpha_2\varepsilon^2+\cdots) \tag{27}$$

and

$$W_+ + W_- = \alpha_0+\alpha_2\varepsilon^2+\cdots, \tag{28}$$

where $p_\pm = \pm 1$. The coefficients of these expansions are given by

$$\frac{1}{2}\alpha_0 \equiv W_\pm(\varepsilon=0)=\frac{1}{\sqrt{\pi}\tau}\sigma^{3/2}\exp(-\sigma) \quad , \tag{29}$$

$$\frac{1}{2}\alpha_1p_\pm \equiv \frac{d}{d\varepsilon}W_\pm(\varepsilon=0)=\frac{p_\pm}{\sqrt{\pi}\tau}\sigma^{3/2}(1+2\sigma)\exp(-\sigma) \quad , \tag{30}$$

and

$$\begin{aligned}\frac{1}{2}\alpha_2p_\pm &\equiv \frac{1}{2}\frac{d^2}{d\varepsilon^2}W_\pm(\varepsilon=0) \\ &= \frac{1}{\sqrt{\pi}\tau}\sigma^{3/2}\exp(-\sigma)(4\sigma^2+2\sigma-2) \quad ,\end{aligned} \tag{31}$$

where we have defined the parameter $\sigma = K_aV_p/k_BT$, comparing anisotropy and thermal energies.

The averaged power spectrum in a period of the input signal is written as

$$\begin{aligned}\overline{S(\omega)} &= \frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0}S(\omega,t)dt \\ &= \frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0}dt\int_{-\infty}^{\infty}du\ \langle x(t)x(t+u)\rangle\exp(-i\omega u) \quad .\end{aligned} \tag{32}$$

Using the solutions (25), the approximations (27) and (28), and the expressions (29)–(31), it is possible to compute the correlation function $\langle x(t)x(t+u)\rangle$ and from it $\overline{S(\omega)}$, which for positive ω reads

$$\overline{S(\omega)} = \left[1 - \frac{2\varepsilon_0^2\sigma^3(1+2\sigma)^2 e^{-2\sigma}}{\pi\tau^2\left(4\frac{\sigma^3 e^{-2\sigma}}{\pi\tau^2}+\omega_0^2\right)}\right]\frac{8\sigma^{3/2}e^{-\sigma}}{\sqrt{\pi}\tau\left(4\frac{\sigma^3 e^{-2\sigma}}{\pi\tau^2}+\omega^2\right)} + \frac{4\varepsilon_0^2\sigma^3(1+2\sigma)^2 e^{-2\sigma}}{\tau^2\left(4\frac{\sigma^3 e^{-2\sigma}}{\pi\tau^2}+\omega_0^2\right)}\delta(\omega-\omega_0)\ . \qquad (33)$$

This equation clearly shows a resonant peak at $\omega = \omega_0$, which indicates that the fluctuations of this mode diverge as a δ function. From Eq. (33) one can compute the signal to noise ratio $I_{\rm SNR}$ as a function of the parameter σ. We obtain

$$I_{\rm SNR}(\sigma) = \frac{\sqrt{\pi}}{2\tau}\left\{\frac{\varepsilon_0^2\sigma^{3/2}(1+2\sigma)^2 e^{-\sigma}}{1-\frac{2\varepsilon_0^2\sigma^3(1+2\sigma)^2 e^{-2\sigma}}{\pi\tau^2\left(4\frac{\sigma^3 e^{-2\sigma}}{\pi\tau^2}+\omega_0^2\right)}}\right\}\ . \qquad (34)$$

In Fig. 2 we have plotted this function for some values of the parameter ε_0.

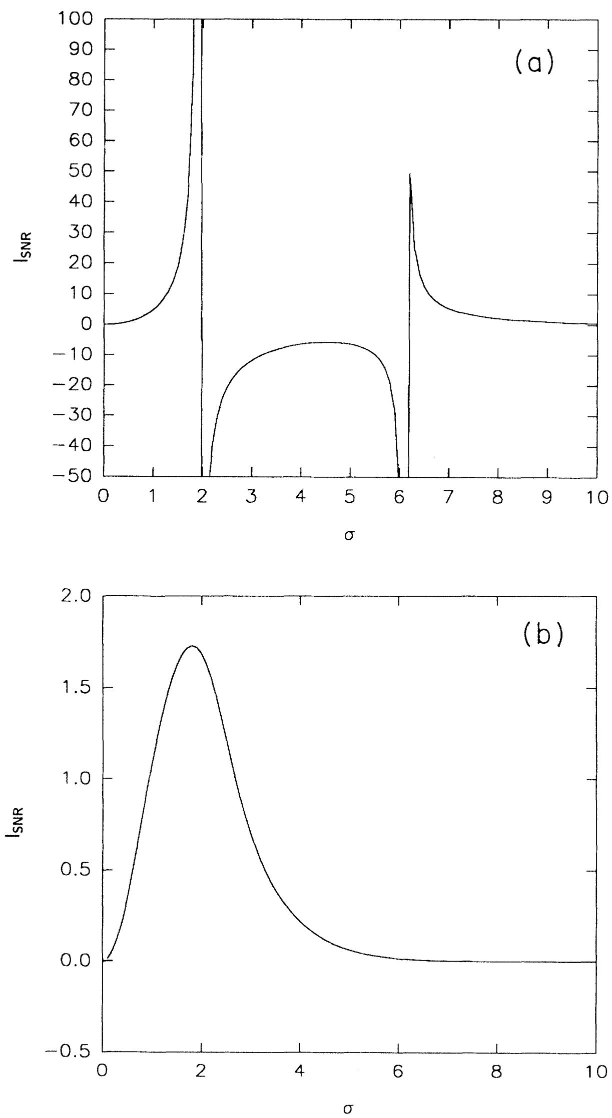

FIG. 2. Signal to noise ratio $I_{\rm SNR}$ in units of $\sqrt{\pi}/2\tau$ as a function of σ (a) for $\varepsilon_0 = 0.3$ and (b) $\varepsilon_0 = 0.1$. In both cases $\omega_0\tau = 0.1$.

IV. DISTRIBUTION OF RESIDENCE TIMES AND MEAN FIRST PASSAGE TIME

In this section we will compute the distribution fuction of the residence times around the states $x_\pm$, which correspond to the wells of the potential. This can be done by assuming the existence of an absorbing barrier between the two minima of the potential energy $U(\theta)$. Then Eq. (19) reduces to

$$\frac{d}{dt}n_\pm = -W_\pm n_\pm\ . \qquad (35)$$

With the initial condition $n_\pm(0) = 1$, the solution of (35) is written as

$$n_\pm(t) = \exp\left[-\frac{1}{\omega_0}\int_0^{\omega_0 t} W_\pm(z)dz\right]\ . \qquad (36)$$

The distribution function for the residence times [16,17] in the well centered at $x_\pm, \rho_\pm(t)$ then reads

$$\rho_\pm(t) = -\frac{d}{dt}n_\pm = W_\pm \exp\left[-\frac{1}{\omega_0}\int_0^{\omega_0 t} W_\pm(z/\omega_0)dz\right]\ . \qquad (37)$$

Up to order ε^2 one has

$$\rho_\pm = \frac{1}{2}\left[\alpha_0 + \alpha_1 p_\pm \varepsilon(t) + \alpha_2\varepsilon^2(t)\right] \times\exp\left[-\frac{1}{\omega_0}\int_0^{\omega_0 t}[\alpha_0 + \alpha_1 p_\pm \varepsilon(z\omega_0) + \alpha_2\varepsilon^2(z\omega_0)]dz\right] \qquad (38)$$

where we have used (27). From (29)–(31) and (38) one finally obtains

$$\rho_{\pm} = \frac{\sigma^{3/2}e^{-\sigma}}{\sqrt{\pi}\tau}\left[1 + (1+2\sigma)p_{\pm}\varepsilon_0\sin(\omega_0 t) + 2(2\sigma^2+\sigma-1)\varepsilon_0^2\sin^2(\omega_0 t)\right]$$
$$\times\exp\left\{-\frac{\sigma^{3/2}e^{-\sigma}}{\sqrt{\pi}\tau\omega_0}\left[\omega_0 t + (1+2\sigma)p_{\pm}\varepsilon_0[1-\cos(\omega_0 t)] + (2\sigma^2+\sigma-1)\varepsilon_0^2[\omega_0 t - \tfrac{1}{2}\sin(2\omega_0 t)]\right]\right\} . \quad (39)$$

As follows from this equation, the input signal generates a coherent response of the system in the sense that the distribution of residence times is modulated by the signal.

Finally, we will proceed to compute the mean first passage time. Notice that in view of its definition $\rho_+ \, dt$ gives us the probability that the magnetic moment reaches the absorbing boundary at θ_m coming from the well +, in the time between t and $t+dt$. Thus it is easy to find the mean first passage time $\langle T\rangle$, which is given by

$$\langle T\rangle = \int_0^\infty dt\, t\rho_+(t) = \int_0^\infty dt\, n_+(t), \quad (40)$$

where we have employed (35) and performed an integration by parts. By using (25)–(27) and (29)–(31) in Eq. (40) one achieves

$$\langle T\rangle = \int_0^\infty dt \exp\left\{-\frac{1}{2\omega_0}\left[\alpha_0\omega_0 t + \alpha_1 p_+\varepsilon_0(1-\cos\omega_0 t) + \frac{1}{2}\alpha_2\varepsilon_0^2(\omega_0 t - \tfrac{1}{2}\sin 2\omega_0 t)\right]\right\} . \quad (41)$$

For $\varepsilon_0 \le 0.1$ we can expand the exponential in terms of this small parameter. Keeping terms of order ε_0^2 and performing the resulting integral our result reads

$$\langle T\rangle/\langle T_0\rangle = 1 - \frac{1}{2}\alpha_1\varepsilon_0\frac{\omega_0}{(\alpha_0/2)^2 + +\omega_0^2} + \frac{1}{8}\alpha_1^2\varepsilon_0^2\frac{6\omega_0^2}{[(\alpha_0/2)^2+4\omega_0^2][(\alpha_0/2)^2+\omega_0^2]} - \alpha_2\varepsilon_0^2\frac{\omega_0^2}{(\alpha_0/2)[(\alpha_0/2)^2+4\omega_0^2]}, \quad (42)$$

where $\langle T_0\rangle$ is the mean first passage time in the absence of the external signal.

V. DISCUSSION

From the results we have obtained in previous sections, the following comments are in order. In Figs. 2(a) and 2(b) we have plotted the quantity $I_{\rm SNR}/(\sqrt{\pi}/2\tau)$, obtained from Eq. (34), for the values $\varepsilon_0 = 0.3$ and $\varepsilon_0 = 0.1$, with $\omega_0\tau$ kept fixed. Note that it should be $\omega_0\tau < 1$ because the time scale when the system achieves the stationary state should be shorter than the period of the signal, essentially ω_0^{-1}. What Fig. 2(a) makes evident is that one must take into account the interplay between the parameters ε_0 and σ in order for the discrete probability approach to be valid. Thus, from this plot we deduce that for $\varepsilon_0 = 0.3$ our result is correct whenever $\sigma \ge 6.09$. The two vertical asymptotic straight lines originate in two zeros of the noise power spectrum (33). In the case of a constant external field, Brown [8] found that the discrete probability approach works fairly well for $\sigma \ge 0.92$. For lower values of ε, as in Fig. 2(b), our hypothesis works in the whole range $\sigma \ge 1$. From Fig. 2(b) we see that $I_{\rm SNR}$ exhibits a maximum at a certain value of the parameter σ. The reason for the appearance of such a maximum follows from the following argument. From (14), (15), and (21)–(24) we can write Eq. (20) as

$$W_+ = \frac{h}{\sqrt{\pi}}m_s H_c(1+\varepsilon-\varepsilon^2-\varepsilon^3)\sigma^{1/2} \times\exp[-\sigma(1+\varepsilon+\varepsilon^2)] . \quad (43)$$

For $\sigma \gg 1$ one has

$$W_+ \sim \frac{h}{\sqrt{\pi}}m_s H_c\sigma^{1/2}e^{-\sigma}, \quad (44)$$

which corresponds to an incoherent response. At a very high temperature, $\sigma = 0$; consequently $W_+ = 0$ and the behavior of the magnetic moment is completely random. Thus there should be some finite value of σ for which the coherence between the response of the system and the signal must be a maximum.

The quantity $d_+ \equiv \rho_+/(\sigma^{3/2}e^{-\sigma}/\sqrt{\pi}\tau)$ has been plotted in Fig. 3 as a function of the dimensionless time $t' \equiv \omega_0 t$. The picture shows a pattern of peaks, each one upon a quarter of the period of the input signal τ_0. Moreover, the minima appear for even multiples of $\tau_0/4$.

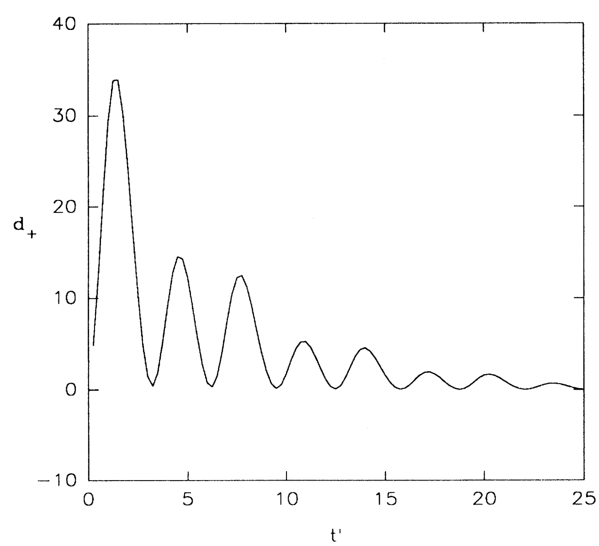

FIG. 3. Plot of $d_+ \equiv \rho_+/(\sigma^{3/2}e^{-\sigma}/\sqrt{\pi}\tau)$ as a function of the nondimensional time $t' = \omega_0 t$, with $\varepsilon_0 = 0.3$, $\omega_0\tau = 0.1$ and $\sigma = 10$.

Notice that the first maximun at $\tau_0/4$ is higher than the second one at $3\tau_0/4$. This fact is in good agreement with the time dependence of the energy observed in Fig. 1. The well at θ_+ is deeper for $t = \tau_0/4$ than for $t = 3\tau_0/4$, which explains the behavior of ρ_+ in the first cycle and so on in the subsequent cycles. For times long compared to τ_0, a decay of the function d_+, and consequently of the distribution ρ_+, is observed. The physical meaning of this behavior is that after a long time all the particles initially present at this well have already gotten through the barrier. Essentially, the main feature of this picture coincides with the corresponding one in Ref. [18].

Finally, with respect to Fig. 4, it is worth to noting that, as has already been reported [17], the mean first passage time decreases drastically, in our case, up to a minimum value. In the absence of modulation this quantity would remain constant.

It is interesting to give some typical values for the parameters appearing in our calculations. From Ref. [19] one has $m_s/V_p = 480$ G s, $V_P = 5\times10^{-19}$ cm^3, and $K_a = 1.9\times10^5$ erg cm^{-3} for magnetite and $m_s/V_p = 1400$ G s, $V_P = 2.7\times10^{-19}$ cm^3, and $K_a = 8\times10^5$ erg cm^{-3} for cobalt. On the other hand, an estimation of the damping parameter appearing in the Gilbert equation can be found by looking for the value that maximizes τ [20,21]; one has $\eta = 1/m_s\gamma$. Then at room temperature, $k_BT = 4\times10^{-14}$ erg, and $\gamma = 2\times10^7$ Oe^{-1} s^{-1} we obtain $\tau = 3.1\times10^{-10}$ s, $\omega_0 < 3.2\times10^9$ s^{-1}, $\sigma = 2.38$, and $H_c = 791.7$ Oe for magnetite and $\tau = 4.8\times10^{-10}$ s, $\omega_0 < 2.1\times10^9$ s^{-1}, $\sigma = 5.4$, and $H_c = 1142.8$ Oe for cobalt. From these data it follows that the range of values of the external inputs H_0 and ω_0 is of experimental accessibility [19]. Moreover, we conclude that the short time scale involved in our system confirms the validity of the adiabatic approximation.

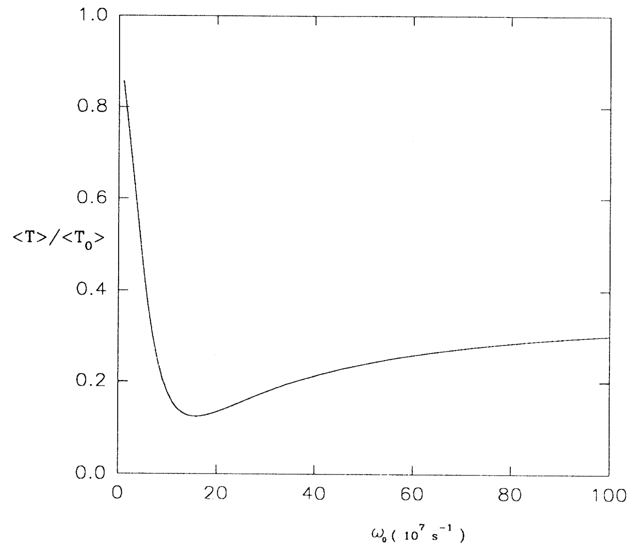

FIG. 4. Normalized mean first passage time $\langle T\rangle/\langle T_0\rangle$ as a function of the angular frequency ω_0 of the external field.

ACKNOWLEDGMENT

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